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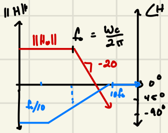
Filters

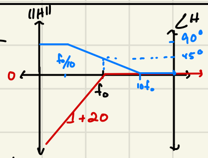
Single pole: $\frac{H_0}{1+\frac{s}{\omega_c}}$



inverted pole: $\frac{1}{1+\frac{\omega_0}{s}}$

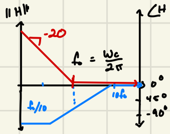
$\frac{1}{1+\frac{\omega_0}{s}} = \frac{(s/\omega_0)}{1+\frac{s}{\omega_0}}$

inverted pole = zero + regular pole

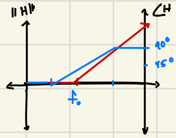


inverted zero: $\left(1+\frac{\omega_z}{s}\right)$

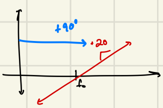
right hand $1-\frac{\omega_z}{s}$ phase flipped



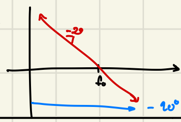
regular zero $\left(1+\frac{s}{\omega_z}\right)$



$\frac{s}{\omega_0}$



$\frac{\omega_0}{s}$



Graphical

if R crosses L : $f = \frac{R}{2\pi L}$

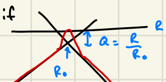
if R crosses C : $f = \frac{1}{2\pi RC}$

if L crosses C : $f = \frac{1}{2\pi\sqrt{LC}}$

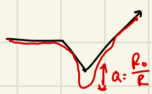
if C crosses axis : $f = \frac{1}{2\pi C}$

$R_0 = \sqrt{\frac{L}{C}}$

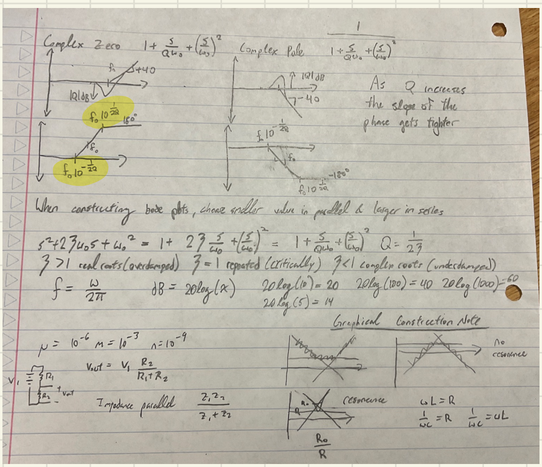
:f



:f



RLC = Q


Complex Zero $1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2$ Complex Pole $\frac{1}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2}$

f_0, +40, $|Q|$ dB, $f_0 10^{\frac{1}{2Q}}$, 180°, f_0, $f_0 10^{-\frac{1}{2Q}}$

$|Q|$ dB, -40, $f\,10^{-\frac{1}{2Q}}$, f_0, $f_0 10^{\frac{1}{2Q}}$, -180°

As Q increases the slope of the phase gets tighter

When constructing bode plots, choose smaller value in parallel & larger in series

$s^2+2\zeta\omega_0 s+\omega_0^2 = 1+2\zeta\frac{s}{\omega_0}+\left(\frac{s}{\omega_0}\right)^2 = 1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2 \quad Q=\frac{1}{2\zeta}$

$\zeta>1$ real roots (overdamped) $\zeta=1$ repeated (critically) $\zeta<1$ complex roots (underdamped)

$f=\frac{\omega}{2\pi}$ dB = 20log(x) 20log(10) = 20 20log(100) = 40 20log(1000) = 60 20log(5) = 14

Graphical Construction Note

$\mu = 10^{-6}$ $m = 10^{-3}$ $n = 10^{-9}$

$V_{out} = V_1\frac{R_2}{R_1+R_2}$

Impedance parallel $\frac{Z_1Z_2}{Z_1+Z_2}$

resonance no resonance

$\omega L = R$ $\frac{1}{\omega C}=R$ $\frac{1}{\omega C}=\omega L$

$\frac{R_0}{R}$

slope = ·180·Q

$G(s) = \frac{V_{out}}{V_{in}} = \frac{Z_{out}}{Z_1}$

Convolution

$x(t) \to \boxed{H(t)} \to y(t)$

impulse response $H(t) = \mathcal{L}^{-1}\{H(s)\}$

$y(t) = \int_0^t \underbrace{h(t-\tau)}_{\text{impulse}}\,\underbrace{x(\tau)}_{\text{input}}\,d\tau$

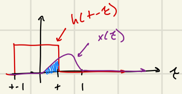
$y(t) = \mathcal{L}^{-1}\{H(s)X(s)\}$

$y(t) = ?$ for $0<t<\infty$

$h(t) = u(t) - u(t-1)$

input $x(t) = \begin{cases} \sin(\pi t), & 0\le t\le 1 \\ 0, & \text{otherwise}\end{cases}$

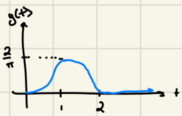
for $0\le t\le 1$



$y = \int_0^t 1(\sin(\pi\tau))\,d\tau$

$y = \frac{1}{\pi}\left(-\cos(\pi\tau)\right)\Big|_0^t$

$y = \frac{1-\cos(\pi t)}{\pi}$



for $1<t<2$

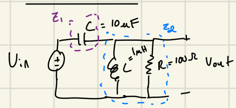
$y(t) = \int_{t-1}^1 1(\sin(\pi\tau))\,d\tau$

$y(t) = \frac{1}{\pi}\left(-\cos(\pi\tau)\right)\Big|_{t-1}^1$

$y(t) = \frac{1}{\pi}\left(1+\cos(\pi(t-1))\right)$

$\|H\|$	$\to$	$\|H\|_{dB}$
1		0 dB
10		20 dB
10^n		n 20 dB
2		~ 6.02 dB
$\sqrt{2}$		3 dB
$1/\sqrt{2}$		-3 dB
5		14 dB

RLC Filter



Solved with Algebra

$G(s) = \frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1+Z_2} = \frac{R\|SL}{1/SC + R\|SL} \Rightarrow$

$\Rightarrow \frac{\left(\frac{RSL}{R+SL}\right)}{\left(\frac{1}{SC}+\frac{RSL}{R+SL}\right)} = \frac{\frac{RS^2LC}{R+SL}}{1+\frac{RS^2LC}{R+SL}} = \frac{RS^2LC}{R+SL+RS^2LC}$

$\omega_0 = \sqrt{\frac{1}{LC}}$

$\Rightarrow \frac{S^2LC}{1+\frac{SL}{R}+S^2LC}$ standard form!! $= \frac{\left(\frac{s}{\omega_0}\right)^2}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2}$

$\frac{L}{R} = \frac{1}{Q\omega_0}$ $Q = \frac{R}{\omega_0 L}$

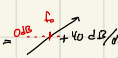
plug in #'s

$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = 1.6\text{ kHz}$

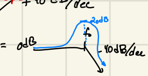
$Q = (100)\sqrt{\frac{10^{-5}}{10^{-3}}} = 10 = 20\text{ dB}$

Construct Asymptotes

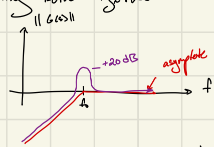
magnitude : numerator : $\left(\frac{s}{\omega_0}\right)^2 =$



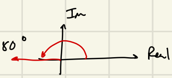
denominator : $\frac{1}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2} =$



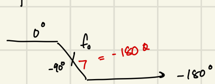
putting both together



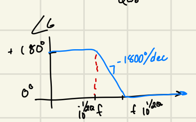
Phase : numerator : $\left(\frac{s}{\omega_0}\right)^2 = +180°$ $s=j\omega$

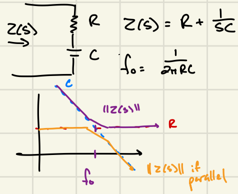


denominator : $\frac{1}{1+\frac{s}{Q\omega_0}+\left(\frac{s}{\omega_0}\right)^2} =$



combine :





$Z(s) = R + \frac{1}{sC}$

$f_0 = \frac{1}{2\pi RC}$

$G(s) = \frac{R_1}{R_2} \cdot \frac{1 + sR_2C}{1 + s\left(\frac{L}{R_1} + R_2C\right) + s^2LC} = G_0 \frac{\left(1 + \frac{s}{w_z}\right)}{1 + \frac{s}{Qw_0} + \left(\frac{s}{w_0}\right)^2}$

$w_z = \frac{1}{R_2C}$

$f = \frac{1}{R_2C(2\pi)}$

$G_0 = \frac{R_1}{R_2} \quad w_0 = \frac{1}{\sqrt{LC}} \quad f_0 = \frac{1}{\sqrt{LC}(2\pi)}$

Improper Rational Function
→ order num. > order den
$\frac{s^2}{s}$ ← use long division
must be proper rational

Residules

roots: $\alpha + \beta i$

Simple Poles

$k_i = (s - p_i) F(s)\big|_{s=p_i}$

Complex Roots → appear in conjugate pairs

$k = |k| e^{j\theta} \quad k^* = |k| e^{-j\theta}$

$\Rightarrow 2|k| e^{-\alpha t}\left(\frac{e^{j(\beta t+\theta)} + e^{-j(\beta t+\theta)}}{2}\right)$

$\Rightarrow 2|k| e^{-\alpha t} \cos(\beta t + \theta)$

$\tan^{-1}\left(\frac{Imag}{Real}\right)$

Ex: multiple poles

$\frac{4(s+3)}{s(s+2)^2}$ or $k_3 = \frac{d}{ds}[k_3]$

$\frac{1}{(s+2)} \frac{4(s+3)}{s(s+2)} = \frac{k_1}{s} + \frac{k_2}{s+2}$

$k_1 = s(F(s))\big|_{s=0} = \frac{4(s+3)}{s+2} = \frac{12}{2} = 6$

$k_2 = (s+2)F(s)\big|_{s=-2} = \frac{4(s+3)}{s} = \frac{4}{-2} = -2$

$\left(\frac{1}{s+2}\right)\left(\frac{6}{s} + \frac{-2}{(s+2)}\right) \rightarrow \frac{6}{(s+2)s} + \frac{-2}{(s+2)^2}$

$\frac{k_1}{(s+2)} + \frac{k_2}{s}$

$k_1 = F(s)(s+2)\big|_{s=-2} = \frac{6}{s} = -3$

$k_2 = \frac{6}{s+2}\big|_{s=0} = 3$

$F(s) = \frac{3}{s} - \frac{3}{s+2} - \frac{2}{(s+2)^2}$

$(3 - 3e^{-2t} - 2te^{-2t})u(t)$

Laplace Transforms

$F(s) = \int_0^\infty f(t) e^{-st} dt$

Time shifted?

$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}F(s)$

Ex: $\mathcal{L}\{\cos(wt)u(t-a)\}$

$\cos(wt - a + a)\, u(t-a)$

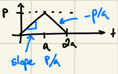
$\cos(w(t-a) + wa)\, u(t-a)$

apply trig identity → $\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\cos(wa)\, e^{-at} \frac{s}{s^2+w^2} + \sin(wa)\, e^{-at} \frac{w}{s^2+w^2}$

Laplace Wave form

Laplace of triangular pulse



$f(t)_1 = (P/a\, t)(u(t) - u(t-a))$

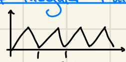
$f(t)_2 = (-P/a\, t + 2P)(u(t-a) - u(t-2a))$

$f(t)_2 = -P/a(t - 2a)(u(t-a) - u(t-2a))$

$\mathcal{L}(f(t)_1 + f(t)_2) = \ldots$

$F(s) = \frac{P}{as^2}(1 - 2e^{-as} + e^{-2as})$

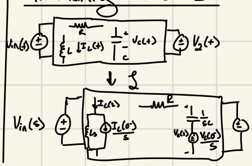
If triangular Pulse Periodic



period = T

$F(s) = \frac{F_1(s)}{1 - e^{-sT}}$ from above $= \frac{P}{as^2}\left(\frac{1 - 2e^{-as} + 2e^{-2as}}{1 - e^{-st}}\right)$

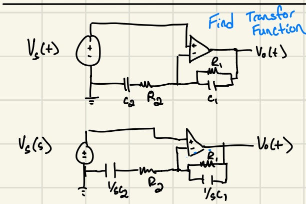
Transforming Circuits



$H(s) = \frac{V_{out}(s)}{V_{in}(s)}$ ← impulse response of circuit

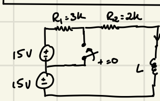
evaluated with all sources not Vin(s) set to 0

Find Transfer Function



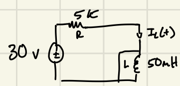
no Vc sources transfer function

Ex:



inductor on for long time = wire

$I_L(0^-) = ?$ $t<0$ switch open

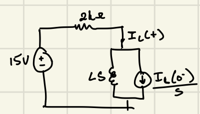


$V = IR$

$I = \frac{V}{R} = \frac{30V}{5000\Omega} = 6mA$

$I_L(0^-) = 6mA$

$t \geq 0$



use superposition
- turn off one all sources but one and do analysis
- do this until all sources have been on
- ⊣⊢ → wire, ⌇⌇⌇ → open
- add terms

15V on

$V = IR \Rightarrow \frac{15}{s} = I(R + Ls)$

$I_1 = \frac{15}{s(R+Ls)}$

I_L on current divider

$\frac{I_L(0^-)}{s}\left(\frac{Ls}{R+Ls}\right)$

$I_L(s) = \frac{15}{s(R+Ls)} + \frac{L(I_L(0^-))}{(R+Ls)} = \frac{15 + (I_L(0^-))Ls}{s(R+Ls)}$

Isolate s terms to ℒ

$\frac{\frac{15}{L} + s(I_L(0^-))}{s(R/L + s)} = I(s)$

$I(t) = \mathcal{L}\{I(s)\}$

USE Partial Fraction

$I(s) = \frac{k_1}{s} + \frac{k_2}{(R/L + s)}$

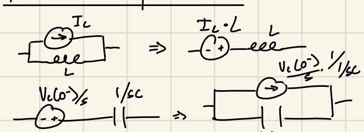
$k_1 = sF(s)\big|_{s=0} \quad \frac{15/L + 0}{R/L} = \frac{15}{2k}$

$k_2 = (R/L + s)F(s)\big|_{s=-\frac{R}{L}} \quad \frac{\frac{15}{L} - \frac{R}{L}(6mA)}{-R/L} = \frac{15}{2k} + 6mA$

$I(t) = \left(\frac{15}{2k} + \left(\frac{15}{2k} - 6mA\right) e^{-R/L\, t}\right) u(t)$

$I(t) = \left(7mA + 1.5mA\, e^{-t/25us}\right) u(t)$

Theveran Equivalence

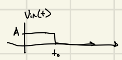


Laplacing Diff Eq

$RC\frac{dv(t)}{dt} + V(t) = Vin(t)$

$V(0) = V_0$

$Vin(t) \rightarrow$



$V(s) = \mathcal{L}\{v(t)\}$

$RC(sV(s) - V_0) + V(s) = A(u(t) - u(t - t_0))$ must ℒ

$\frac{A}{s} - \frac{Ae^{-t_0 s}}{s}$

$V(s)(RCs + 1) = \frac{A}{s}(1 - e^{-t_0 s}) + RCV_0$

$V(s) = \frac{\frac{A}{s}(1 - e^{-t_0 s}) + RCV_0}{RCs + 1}$

make sure to simplify to $\frac{1}{2} + \frac{1}{4}i$

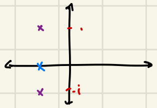
$\frac{2i + 1}{4i}$

Plotting Poles

find roots. Plot Roots

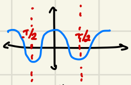
$F(s) = \frac{2s}{(s+2)(s^2+4s+5)}$

roots: -2, $-2 \pm i$



Wave Symitries Fourier

Even: $v(t) = v(-t)$



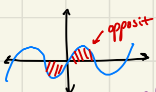
$a_n = \frac{4}{T}\int_0^{T/2} v(t)\cos(n\omega t)\,dt$

$a_0 = \frac{2}{T}\int_0^{T/2} v(t)\,dt$

$b_n = 0$

C_n: only Real

Odd: $v(-t) = -v(t)$



C_n: only Img

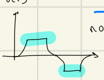
$a_0 = 0$

$a_n = 0$

$b_n = \frac{4}{T}\int_0^{T/2} v(t)\sin(n\omega t)\,dt$

Half wave:

no dc component



no even harmonics ..

$\left.\begin{matrix} a_n = 0 \\ b_n = 0 \end{matrix}\right\}$ for $n = 0, 2, 4,$

$a_n = \begin{cases} 0, & \text{even } n \\ \frac{4}{T}\int_0^{T/2} v(t)\cos(n\omega t)\,dt, & \text{odd } n \end{cases}$

$b_n = \begin{cases} 0, & \text{even } n \\ \frac{4}{T}\int_0^{T/2} v(t)\sin(n\omega t)\,dt, & \text{odd } n \end{cases}$

Fourier Transform of Wave

$C_n = \frac{1}{T} V_1(j\omega n) = \frac{1}{T}\int_0^T v(t)\left(e^{-jn\omega t}\right)dt$

$\omega = \frac{2\pi}{T}$ $= \frac{1}{T} V_1(s)\Big|_{s \to j\omega n}$

$v(t) = a_0 + \sum_{n=1}^{\infty} b_n \sin(n\omega t) + a_n \cos(n\omega t)$

$C_n = \frac{a_n - jb_n}{2}$

when $a_n = 0$ $\quad b_n = 2jC_n$

when $b_n = 0$ $\quad a_n = 2C_n$

$v_1 \to \boxed{G(s)} \to v_2$

cuttoff = 5kHz
critically damped
second order
highpass
$G_0 = 20\,dB$

$G(s) = \frac{G_0}{1 + \frac{\omega_0}{Qs} + \left(\frac{\omega_0}{s}\right)^2}$

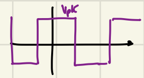
$\omega_0 = (5000)2\pi$

$\omega_0 = 31\,kHz$

$G_0 = 10$

$\frac{1}{Q} = 2\zeta$

$Q = \frac{1}{2}$



$v_1 =$

500 Hz wave

$v_{in}(t) = \sum_{n=1,3,5\ldots}^{\infty} a_n\cos(n\omega t) + b_n\sin(n\omega t)$

$\Rightarrow v_{in}(t) = \sum_{n=1,3,5,\ldots}^{\infty} A_n\cos(n\omega t + \theta_n)$

$a_n = \frac{\sqrt{2}\,V_{pk}}{2\pi n} = b_n$

$V_{pk} = 5V$

$A_n = \sqrt{a_n^2 + b_n^2} = \frac{V_{pk}}{n\pi}$ $\quad \theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right) = -45°$

$\|G(s)\| = \left\|\frac{G_0}{1 + \frac{\omega_0}{Qn\omega j} - \frac{\omega_0^2}{n^2\omega^2}}\right\| = \frac{G_0}{\sqrt{\left(1 - \left(\frac{\omega_0}{n\omega}\right)^2\right)^2 + \left(\frac{\omega_0}{n\omega Q}\right)^2}}$

$\angle G(s) = \tan^{-1}\left(\frac{\omega_0}{n\omega Q}\cdot\frac{1}{1 - \left(\frac{\omega_0}{n\omega}\right)^2}\right)$

$V_{out}(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{5}{n\pi}\|G(s)\|\cos\left(n\omega t - \frac{\pi}{4} - \angle G(s)\right)$

magnitude of 5^{th} hermonic?

$n = 5$ plug in values + solve

Fourier Series Thru Transfer function

$V_{out} = \sum_{n=1,3,5,\ldots} G_0 a_0 + A_n \|G(jn\omega)\|\cos(n\omega t + \theta_n + \angle G(jn\omega))$

polar form

Finding Fourier Coeficients

$a_0 = \frac{1}{T}\int_0^T v(t)\,dt$ (average)

$a_n = \frac{2}{T}\int_0^T v(t)\cos(n\omega t)\,dt$

$b_n = \frac{2}{T}\int_0^T v(t)\sin(n\omega t)\,dt$

C_n Purely Real or Img? Zero for even n?

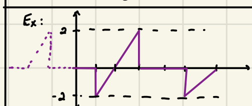
odd symitry
$a_n = 0$
purely imaginary
no half wave
not zero for even n

$v(t)$ in rectangular form

$\sum_{n=1}^{\infty} b_n \sin(n\omega t)$

$b_n = 2jC_n$

Ex:



$T = 4$

$v(t) = 2(t-2)(u(t-1) - u(t-3))$

$v(t) = 2((t-1)-1)u(t-1) - 2((t-3)+1)u(t-3)$

$v(t) = 2(t-1)u(t-1) - 2u(t-1) - 2(t-3)u(t-3) + 2u(t-3)$

$\mathcal{L}(v(t)) = \frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s} - \frac{2e^{-3s}}{s^2} + \frac{2e^{-3s}}{s}$

$= \frac{2e^{-s}}{s}\left(\frac{1}{s} - 1 - \frac{e^{-2s}}{s} + e^{-2s}\right)$

$V(s) = \frac{V_1(s)}{1 - e^{-sT}} = \frac{\frac{2e^{-s}}{s}\left(\frac{1}{s} - 1 - \frac{e^{-2s}}{s} + e^{-2s}\right)}{1 - e^{-4s}}$

Fourier

$C_n = \frac{1}{T} V_1(s)\Big|_{s \to j\omega n}$ $\quad \omega = \frac{2\pi}{T} = \frac{\pi}{2}$

$\Rightarrow C_n = \frac{2e^{-j\omega n}}{jn\omega T}\left(\frac{1}{jn\omega} - 1 - \frac{e^{-2jn\omega}}{jn\omega} + e^{-2jn\omega}\right)$

Forms of Fourier

Rectangular:

a_0 = average DC value

$v(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\omega t) + b_n\sin(n\omega t)$

Polar:

$A_n = \sqrt{a_n^2 + b_n^2} = 2\|C_n\|$ $\quad \theta_n = -\tan^{-1}\left(\frac{b_n}{a_n}\right) = \angle C_n$

$v(t) = a_0 + \sum_{n=1}^{\infty} A_n\cos(n\omega t + \theta_n)$

Euler: Exponential

$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$

High pass filter: $\frac{\omega_0}{s}$

Low pass filter: $\frac{s}{\omega_0}$

Second Order Systems

$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} = \frac{1}{1 + 2\zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$

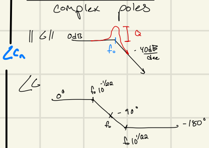
low pass

Critically Damped $\quad Q = \frac{1}{2}$

$\boxed{Q > 1/2 \;\equiv\; \zeta < 1}$

complex poles

$\frac{1}{Q} = 2\zeta$

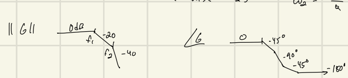


$\boxed{Q < 1/2 \;\equiv\; \zeta > 1}$

2 Real Poles

$G(s)$ will factor to $\frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$

low Q approx:
$\omega_1 \approx Q\omega_0$
$\omega_2 \approx \frac{\omega_0}{Q}$



Butterworth filter

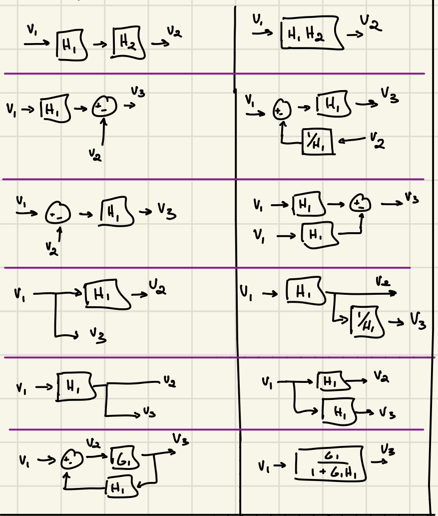
Always Img

3rd order Ex: $\frac{1}{\left(1 + \frac{s}{\omega_c}\right)\left(1 + \frac{s}{\omega_c} + \left(\frac{s}{\omega_c}\right)^2\right)}$

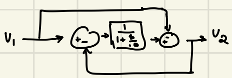
Chebyshev

Block Diagrams

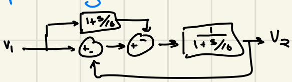
Rules:



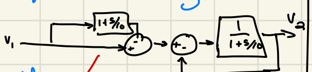
Ex:



push summing nodes to left



swap order of summing nodes

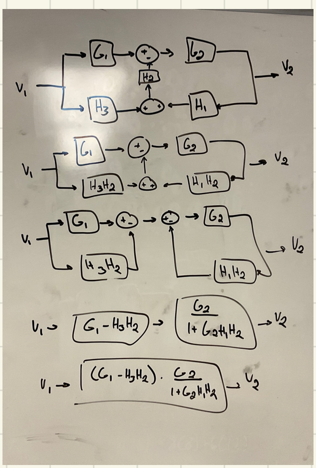


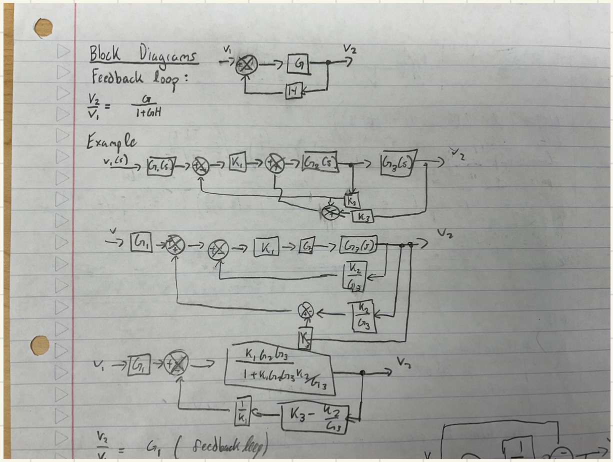
simplify

$$V_1 \rightarrow \boxed{1-\left(1+\frac{s}{10}\right)} \rightarrow \boxed{\frac{\frac{1}{1+s/10}}{1+\frac{1}{1+s/10}}} \rightarrow V_2$$

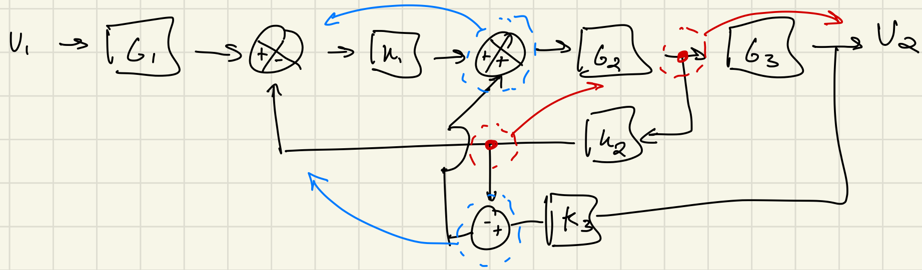
multiply blocks

$$V_1 \rightarrow \boxed{\frac{-s}{s+20}} \rightarrow V_2$$

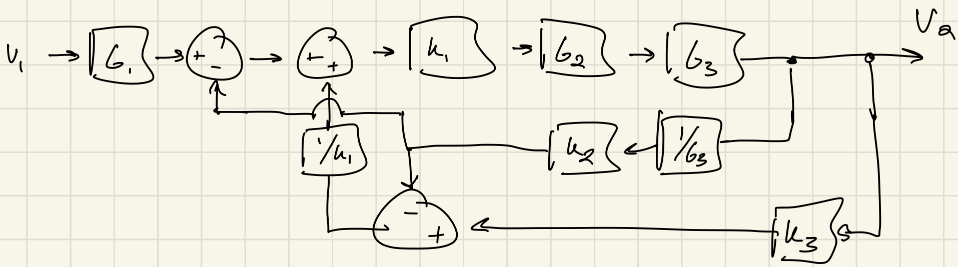


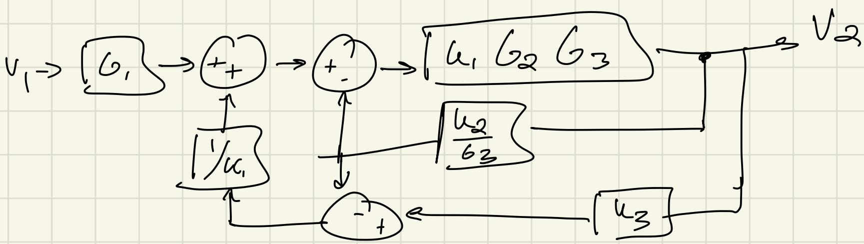


Ex:

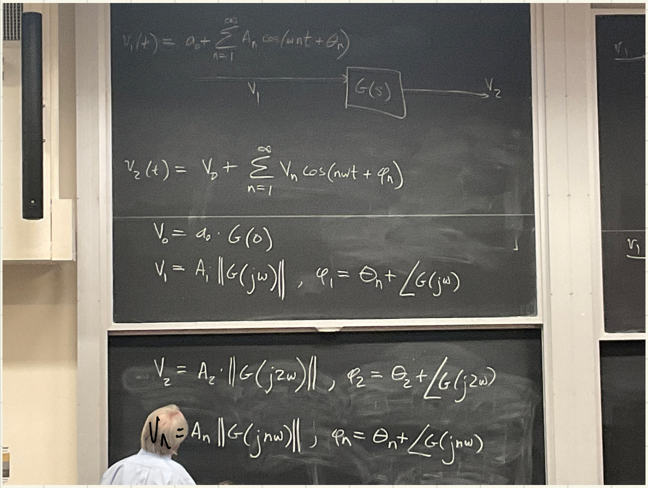


pickoff points to right side
suming nods to left side





$$V_1 \rightarrow \boxed{\frac{K_1 G_1 G_2 G_3}{1 + K_1 K_2 G_2 + G_2 K_2 - G_2 G_3 K_3}} \rightarrow V_2$$



$$v_2(t) = a_0\, G(0) + \sum_{n=1}^{\infty} A_n \, \| G(jn\omega) \| \cos\left(n\omega t + \theta_n + \angle G(jn\omega) \right)$$

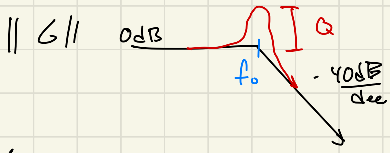
Second Order Systems

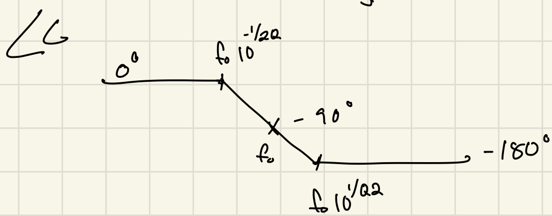
$$\frac{1}{Q} = 2\zeta$$

$$G(s) = \frac{1}{1 + \frac{s}{Q\omega_0} + \left(\frac{s}{\omega_0}\right)^2} = \frac{1}{1 + 2\zeta\frac{s}{\omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\boxed{Q > 1/2 \quad \equiv \quad \zeta < 1}$$

complex poles





$$\boxed{Q < 1/2 \quad \equiv \quad \zeta > 1}$$

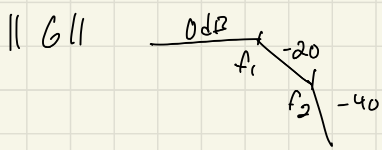
2 Real Poles

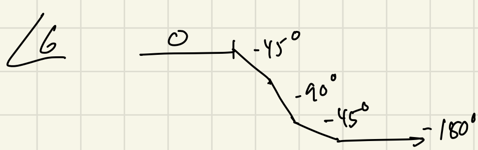
$G(s)$ will factor to $\dfrac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$

low Q approx:

$\omega_1 \approx Q\omega_0$

$\omega_2 \approx \dfrac{\omega_0}{Q}$





Forms of Fourier Series

Rectangular:

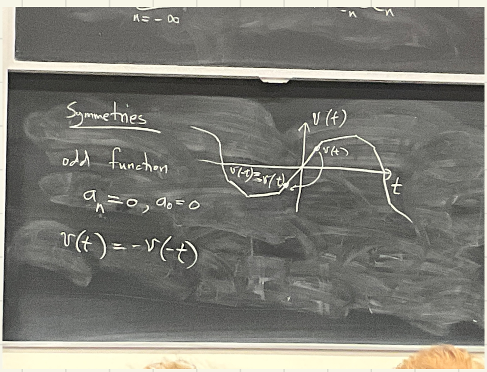
$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

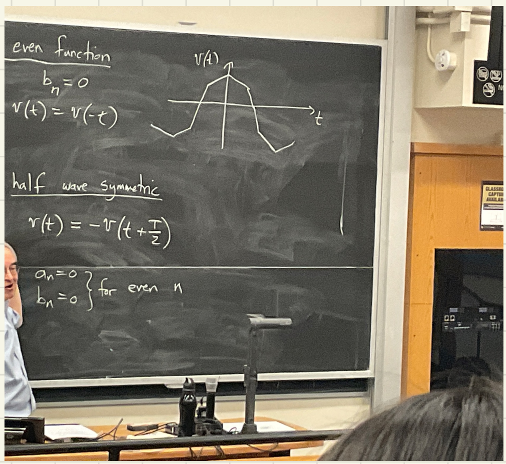
Polar:

$$v(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t + \theta_n)$$

Complex:

$$v(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \qquad \text{where} \quad C_{-n} = C_n^*$$





Final Exam 1

① a) $F(s) = \dfrac{2s}{(s+2)(s^2+4s+5)}$

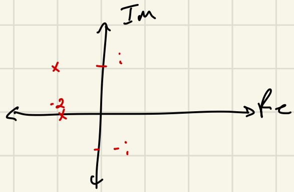
$s=-2$

$\dfrac{-4 \pm \sqrt{16-20}}{2}$

$\dfrac{-4 \pm 2i}{2}$

$= -2 \pm i$

$\alpha \pm \beta$



partla fraction

$F(s) = \dfrac{k_1}{s+2} + \dfrac{k_2}{s-(-2+i)} + \dfrac{k_3}{s-(-2-i)}$

$k_1 = (s+2)F(s)\Big|_{s\to -2} = \dfrac{2s}{s^2+4s+5} = \dfrac{-4}{4-8+5} = -\dfrac{4}{1} = -4$

$k_2 = (s+2-i)F(s)\Big|_{s\to -2+i} = \dfrac{2s}{(s+2)(s-(-2-i))} = \dfrac{2(-2+i)}{(-2+i+2)(-2+i+2+i)} = \dfrac{-4+2i}{i(2i)}$

$\dfrac{-4+2i}{-2}$

$2-i$

$\|k_2\| = \sqrt{4+1}$

$\sqrt{5}$

$\angle k_2 = \tan^{-1}\left(\dfrac{-1}{2}\right) = -45°$

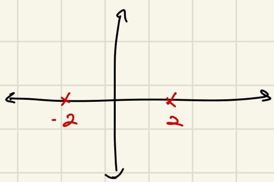
$F(s) = \dfrac{-4}{s+2} + \dfrac{2-i}{s-(-2+i)}$

$\mathcal{L}^{-1}\{\ \}$

$f(t) = -4e^{-2t} + 2\sqrt{5}\,e^{-2t}\cos\left(t - \frac{\pi}{4}\right)$

$F(s) = \dfrac{2s}{(s^2-4)(s+2)}$

roots: -2, ± 2



$F(s) = \dfrac{2s}{(s+2)^2(s-2)}$

$\dfrac{1}{(s+2)} \cdot \dfrac{2s}{(s+2)(s-2)}$

$F(s) = \dfrac{1}{s+2} \cdot \dfrac{k_1}{(s+2)} + \dfrac{k_2}{(s-2)}$

$k_1 = \dfrac{2s}{(s-2)}\Big|_{s\to -2} = \dfrac{-4}{-4} = 1$

$k_2 = \dfrac{2s}{s+2}\Big|_{s\to 2} = \dfrac{4}{4} = 1$

$F(s) = \dfrac{1}{s+2}\left(\dfrac{1}{s+2} + \dfrac{1}{s-2}\right)$

$\dfrac{1}{(s+2)^2} + \dfrac{1}{(s+2)(s-2)}$

$\dfrac{1}{(s+2)^2} + \dfrac{1}{4(s-2)} - \dfrac{1}{4(s+2)}$

$\dfrac{k_1}{s+2} + \dfrac{k_2}{s-2}$

$k_1 = \dfrac{1}{(s-2)}\Big|_{s\to -2} = -\dfrac{1}{4}$

$k_2 = \dfrac{1}{s+2}\Big|_{s\to 2} = \dfrac{1}{4}$

$\Rightarrow f(s) = te^{-2t} + \dfrac{e^{2t}}{4} - \dfrac{e^{-2t}}{4}$

② $v(t) = (2(t-2))(u(t-1) - u(t-3))$ Laplace

a) period = 4

$v_1(t) = 2(t-2)u(t-1) - 2(t-2)u(t-3)$

$\mathcal{L}(v(t)) = 2(t-1-1) - 2(t-3+1)$

$(2(t-1)-2)u(t-1) - (2(t-3)+2)u(t-3)$

$\frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s} - \frac{2e^{-3s}}{s^2} + \frac{2e^{-3s}}{s}$

$$V(t) = \frac{\frac{2e^{-s}}{s^2} - \frac{2e^{-s}}{s} - \frac{2e^{-3s}}{s^2} + \frac{2e^{3s}}{s}}{1 - e^{-4s}}$$

b) $C_n = \frac{1}{T} V_1(j\omega n)$ $s = j\omega n$???

$V_1(s) = \frac{2}{s} e^{-s} \left(\frac{1}{s} - 1 - \frac{1}{s} e^{-2s} - e^{-2s} \right)$

$\frac{2}{j\omega n T} e^{-j\omega n} \left(\frac{1}{j\omega n} - 1 - \frac{e^{-2j\omega n}}{j\omega n} - e^{-2j\omega n} \right)$

d) $v(t) = \cancel{a_0} + \sum_{n=1}^{\infty} \cancel{a_n \cos(n\omega_0)}^{0} + b_n(\sin(jn\omega)$

$v(t) = \sum_{n=1}^{\infty}$

$b_n = 2jC_n$

(3) $V_{in} \rightarrow \boxed{G(s)} \rightarrow V_{out}$

Second order
high pass

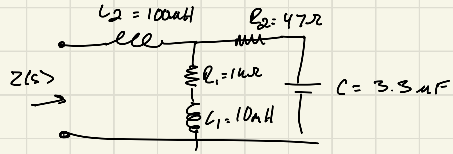
$f_c = 5kHz$

criticly damped

20 dB @ high freq

$$G(s) = \frac{G_0 \checkmark \; = 10 \rightarrow 20dB}{1 + \frac{\omega_0}{sQ} + \left(\frac{\omega_0}{s}\right)^2} \qquad \omega_0 = \frac{5000}{2\pi}$$

(4)



$$L_2 + (R_2 \| C) \| (R_1 \| L_1)$$

$$L_2 S + \frac{1}{\frac{1}{R_2} + SC} \;\Big\|\; \frac{1}{\frac{1}{R_1} + \frac{1}{L_1 S}}$$

$$L_2 S + \frac{R_2}{1 + SCR_2} \;\Big\|\; \frac{R_1 L_1 S}{L_1 S + R_1}$$

$$L_2 S + \frac{1}{\frac{1 + SCR_2}{R_2} + \frac{L_1 S + R_1}{R_1 L_1 S}}$$

$$L_2 S + \frac{R_2 R_1 L_1 S}{(R_1 L_1 S)(1 + SCR_2) + (L_1 S + R_1) R_2}$$

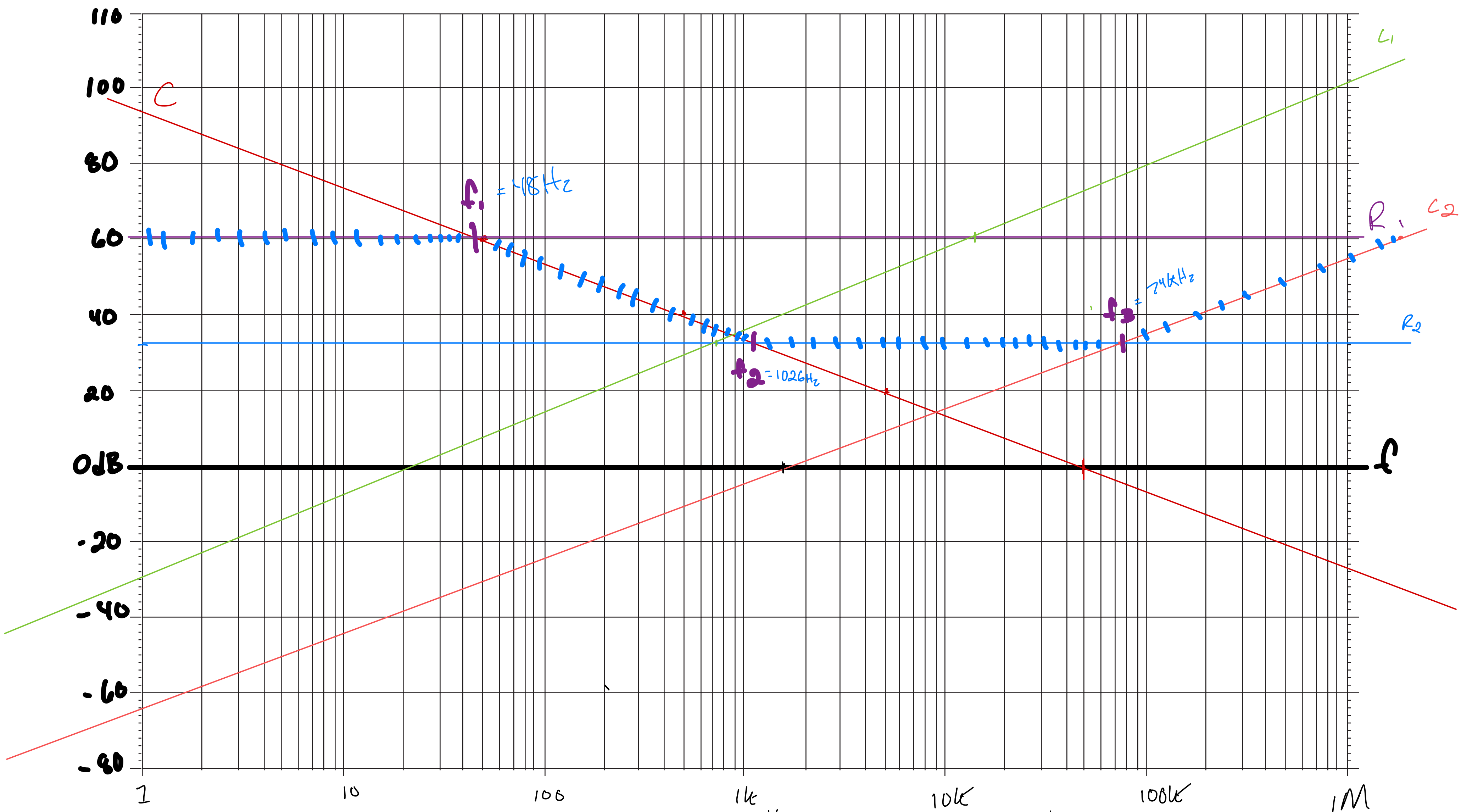
$L_2 + (R_1 + L_1) \parallel (R_2 + C)$

$R_2 = 47\Omega = 33\,dB$

$R_1 = 1k\Omega = 60\,dB$

Good

6 cycle semilog axes



C crosses axis @ $\frac{1}{2\pi\, 3.3\times10^{-6}} = 48.228\,kHz$

R_2 crosses C @ $\frac{1}{2\pi R_2 C} = 1026\,Hz = f_1$

L_2 crosses R_1 @ $\frac{1k}{2\pi \cdot 100\times10^{-6}} = 1.5\,Mh$

↳ $R_2 = 74\,kHz$

Axis @ $\frac{1}{2\pi} = 1.6\,\Omega$

L_1 crosses R_1 @ $\frac{1k}{2\pi \cdot 10\times10^{-3}} = 15915 \approx 15\,kHz$

↳ R_2 @ 748 Hz

(5) $G(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_2 + \frac{1}{sC}}{(R_2 + \frac{1}{sC}) + (Ls + R_1)} = \frac{R_2 sC + 1}{(R_2 sC + 1) + sC(Ls + R_1)} = \frac{R_2 sC + 1}{R_2 sC + 1 + s^2 CL + R_1 sC} = \frac{R_2 sC + 1}{(R_1 + R_2)sC + s^2 CL + 1}$

zero: $R_2 sC + 1 = \frac{1}{R_2 C 2\pi} \div 34\,Hz = f_1$

pole: $(R_1 + R_2) sC + s^2 CL + 1$

$s^2 CL = \left(\frac{s}{\omega_0}\right)^2 = CL = \frac{1}{\omega_0^2} \quad \omega_0 = \sqrt{\frac{1}{LC}}$

$3162k = \omega_0$

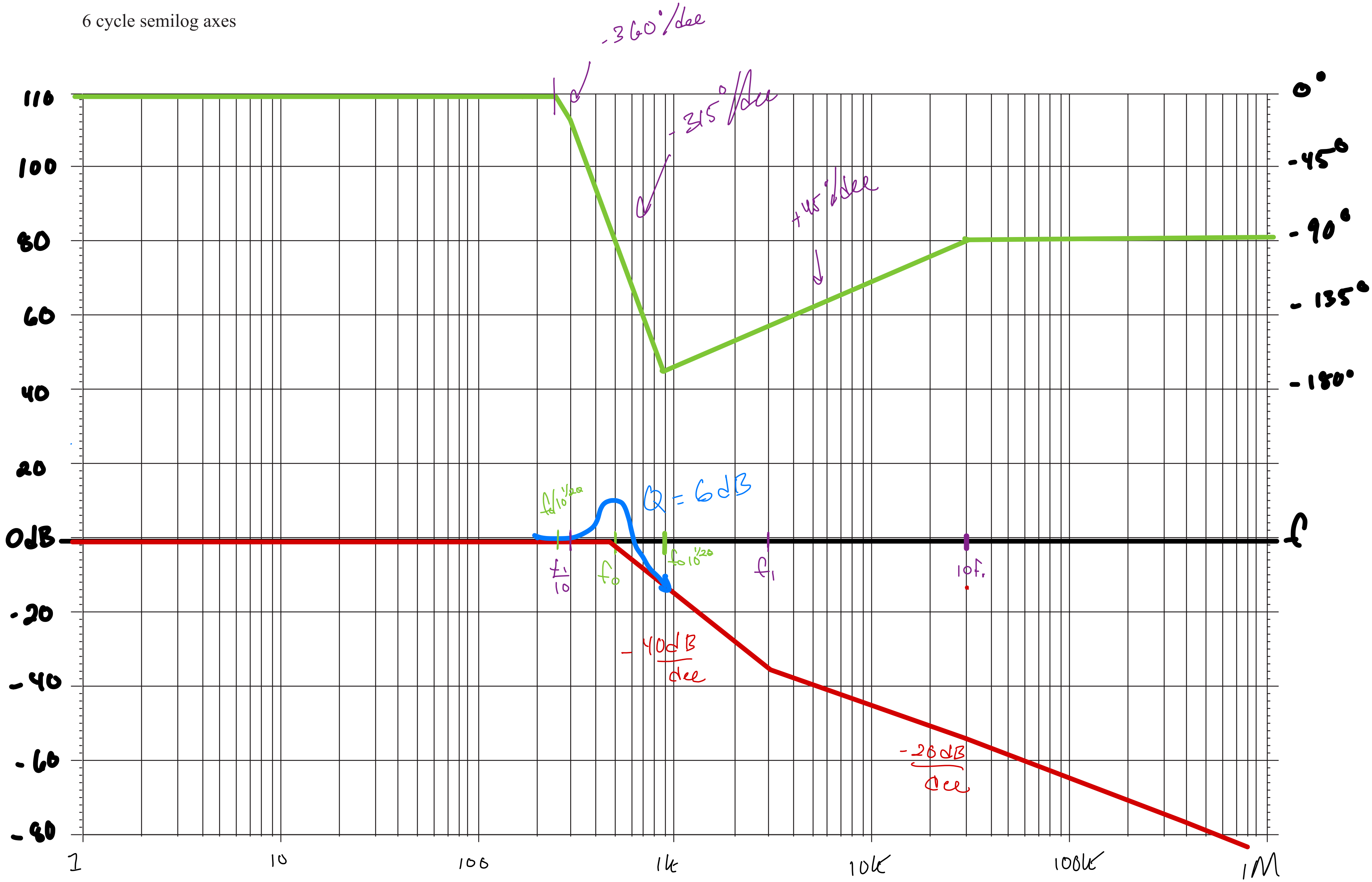
$f_0 = 503\ Hz$

$\frac{1}{Q(3.16k)} = (R_1 + R_2)C$

$Q = \frac{1}{(R_1 + R_2)C(3.16k)}$

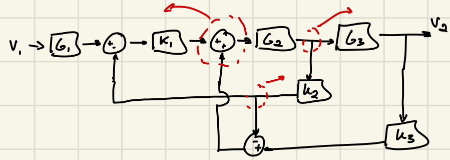
$Q = 2.06$

6 cycle semilog axes

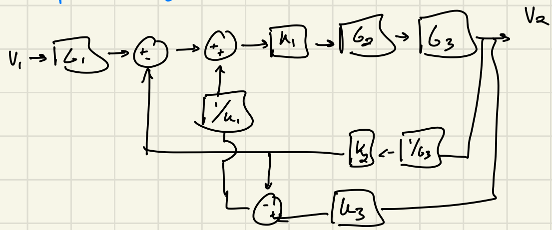


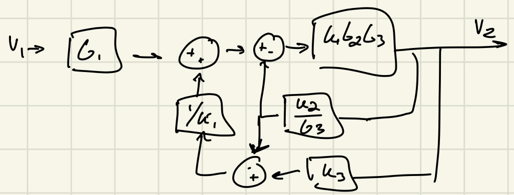
$$\frac{f_0}{10^{1/20}} = 281 \qquad f_0 \cdot 10^{1/20}$$

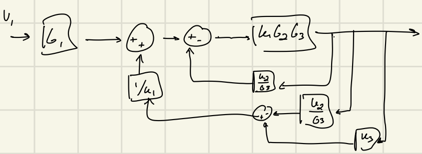
⑥

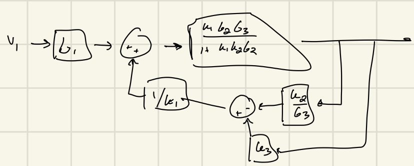


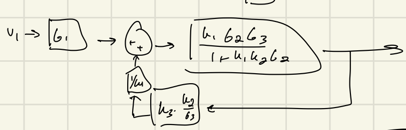
push nodes to left
push points to right

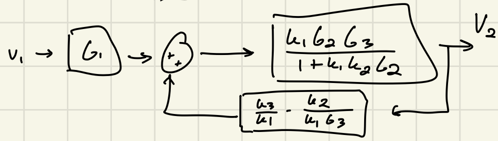








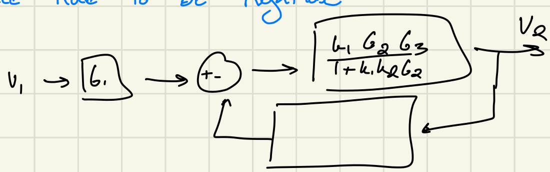




$$\frac{V_1}{V_2} = \boxed{G_1 \cdot \frac{\frac{k_1 G_2 G_3}{1 + k_1 k_2 G_2}}{1 + \left(\frac{\cancel{k_1} G_2 \cancel{G_3}}{1 + k_1 k_2 G_2}\right) \cdot \left(\frac{k_2}{\cancel{k_1 G_3}} - \frac{G_3 k_3}{\cancel{k_1}}\right)}}$$

$$G_1 \cdot \frac{\frac{k_1 G_2 G_3}{1 + k_1 k_2 G_2}}{1 + \frac{G_2 \cdot (k_2 - G_3 \cdot k_3)}{1 + k_1 k_2 G_2}}$$

force node to be negative



HW 10

a) even

$$4 \int^{T/2} v(t)\cos(n\omega t)\,dt$$

$A_n = a_n$

$\theta_n = \tan^{-1}\left(\frac{0}{a_n}\right)$

$= 0$

b) odd symitcy

$a_0 = 0$

$a_n = 0$

$b_n = \frac{4}{T}\int_0^{T/2} v(t)\sin(n\omega t)\,dt$

$$\frac{4}{T}\int_0^{T/2} \frac{I_{pk}\cdot T}{2} t \sin(n\omega t)$$

$A_n = b_n$ $\theta_n = \tan^{-1}\left(\frac{b_n}{0}\right) = \pi/4$

c) even half wave

2 a) Fourier Series of function

$T = 22$ even half wave

$$\left.\begin{array}{l} a_n = 0 \\ b_n = 0 \end{array}\right\} \text{ for } n = 0, 2, 4$$

$$a_n = \begin{cases} 0 & \text{, even } n \\ \frac{4}{T}\int_0^{T/2} v(t)\cos(n\omega t)\,dt & \text{, odd } n \end{cases}$$

$$b_n = 0$$

$$a_n = \frac{4}{22}\int_0^{11}$$